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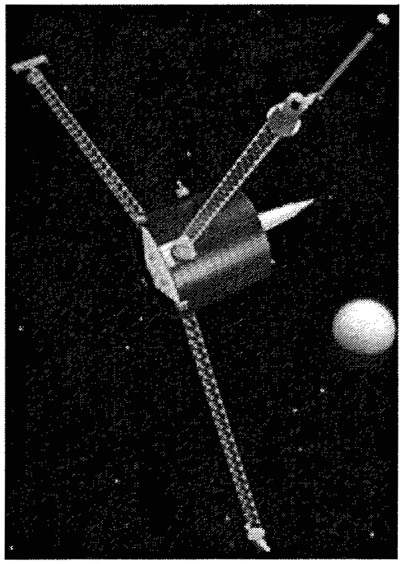

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Lunar Prospector Orbit Determination Uncertainties Using the High Resolution Lunar Gravity Models

Eric Carranza*, **Alex Konopliv***, **Mark Ryne***

Orbit determination of the Lunar Prospector spacecraft performed at the Jet Propulsion Laboratory was conducted as part of the lunar gravity experiment. The orbit determination was performed using S-band two-way Doppler and range data from the Deep Space Network stations with near continuous tracking. A description of the mission and its orbit will be provided, followed by a discussion of the orbit determination estimation procedure and models. Accuracies will be examined in terms of orbit-to-orbit solution differences for several recent gravity models. The orbit determination accuracies for the latest 100th degree gravity model (LP100J) are better than one meter radially for the nominal mission but are substantially higher for the lower altitude orbits of the extended mission.

INTRODUCTION

The Lunar Prospector (LP) spacecraft was launched from Cape Canaveral, FL on January 6, 1998 aboard Lockheed Martin's three-stage solid-fuel Athena II rocket. It was NASA's third Discovery mission to be developed and the second to complete its mission successfully. The objectives of the mission were to generate a global compositional map of the lunar surface, detect lunar outgassing, to determine whether water ice exists at the lunar poles, and improve knowledge of the lunar magnetic and gravity fields. On January 15, 1998, the spin-stabilized LP was placed in a polar, near circular mapping orbit with a period of 118 minutes and an altitude of approximately 100-km. This trajectory provided global coverage of the lunar surface for science collection every two weeks. During the one year primary mission, LP executed orbit correction and reorientation maneuvers to maintain its desired orbit and attitude. On December 19, 1998, the altitude of LP was reduced to an average of 40-km to better determine the lunar gravity field in preparation for the extended mission. Lunar Prospector began its extended mission on January 29, 1999, when the spacecraft was lowered to an altitude of 30-km above the lunar surface to obtain higher resolution mapping data. The LP mission ended July 31, 1999 when it was attempted to crash the spacecraft into a permanently shadowed crater located in the Moon's south pole region in hopes of freeing a water vapor plume from the lunar surface that could be viewed from Earth.

Orbit determination of LP performed at the Jet Propulsion Laboratory (JPL) was conducted as part of the science investigation of the lunar gravity field and this paper describes the effort in support of the gravity experiment. This support includes high precision orbit determination, gravity model validation, and data editing. This work, especially the orbit determination accuracies, is of interest for future missions to the Moon;

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such as, Selene, Lunar-A, European proposals like Moro, future Discovery proposals, and private adventures.

The navigation of LP was performed by the Guidance, Navigation, and Control Center (GNCC) at Goddard Space Flight Center using models for the lunar gravity field developed at JPL using LP tracking data and historical data from Lunar Orbiters I-V, Apollo 15 and 16 subsatellites, and Clementine. Orbit determination results have been presented[1] for the initial gravity models from LP (LP75A and LP75D). In addition, LP was passively tracked by the German Space Flight Operations Center (GSOC) and orbit determination results have been presented[2] using their preliminary gravity determination effort and the more recent 75th degree and order model LP75G from JPL[3]. Our results here also use the LP75G model which is based upon the first two months of LP tracking and the latest 100th degree model LP100J[4] that includes all the nominal mission data and the low altitude tracking data from Dec. 19, 1998 to Feb. 8, 1999.

SPACECRAFT DESCRIPTION

The spacecraft was built by Lockheed Martin Missiles & Space of Sunnyvale, CA and the mission was managed by the NASA Ames Research Center[5]. It was shaped like a cylinder, about 1.3 m in height and 1.4 m in diameter. The spacecraft weighed 295 kg when fully fueled, of which 158 kg was dry mass and 137 kg was propellant; after lunar orbit insertions to a circular orbit, 33 kg of propellant remained and 14 kg of this fuel was used during the 100-km mapping orbit[6]. LP had surface mounted solar cells used to recharge the nickel-hydrogen batteries, which powered the spacecraft and its instruments. It had three booms (also called masts) equally spaced and anchored on its side; each mast extended out 2.4 m (see Fig. S-1). These masts isolated the five instruments LP carried from the rest of the spacecraft body. The instruments were the Gamma Ray Spectrometer (GRS), the Alpha Particle Spectrometer (APS), the Neutron Spectrometer (NS), the Magnetometer (Mag), and the Electron Reflectometer (ER). The GRS was used to search for ten specific elements (O, Mg, Si, Al, K, Ca, Ti, Fe, Th, & U) on the Moon to create a global map of those elements. The APS searched for Moon outgassing, while the NS searched for water ice. The Mag measured the magnetic fields at the spacecraft of the Sun, Earth, and Moon, and the ER remotely measured the magnetic field at the surface of the Moon. The instrument and engineering data was down-linked continuously to the Deep Space Network (DSN) 26 meter and 34 meter diameter tracking stations with a 3.6 kbs telemetry rate.

The Doppler Gravity Experiment (DGE) did not collect any data aboard the spacecraft but used an S-band Loral-Conic transponder for coherent two-way Doppler measurement with the DSN. In addition, ranging data were collected. LP used two antennae to communicate with the Earth. An omni antenna was always used for the up-link signal from the DSN to the spacecraft and sometimes used for the down-link from the spacecraft to the DSN. However, a medium-gain antenna was predominantly used for the down-link. Both antennae were fixed on top of the bus. The up-link S-band frequency was 2093 Mhz and the down-link frequency was 2273 Mhz (240/221 x up-link frequency). By studying the Doppler effect, the Moon's gravity field was directly calculated for the nearside but was only indirectly measured for the farside since no communication could be made with the spacecraft while it was occulted by the Moon. LP was spin-stabilized at approximately 12 rpm with 0.1 rpm for an allowed drift value. The spin-axis was pointed to the ecliptic north within 10° for the first nine months of the mission and then was switched to the ecliptic south for the remainder of the mission[6].

ORBIT HISTORY

The lunar gravity field has a strong effect on the evolution of a low altitude (100-km) near circular orbit. Orbit maintenance maneuvers will usually be required to keep the spacecraft within a desired altitude range (e.g. 80 to 120 km for LP). A drastic example of gravity perturbations on an orbit is the Apollo 16 subsatellite. A simple spin-stabilized fields and particles satellite similar in shape to LP, it was released from the Command Service Module in a near circular 100-km orbit with a 10° inclination to the lunar equator. Since the subsatellite did not have a propulsion system, its altitude could not be raised and the spacecraft impacted the lunar surface 35 days after it was released. Its impact was strictly due to the gravity field. The long term altitude trend of any circular orbit at a given altitude is a function of the orbit inclination and zonal coefficients of the gravity field and so any other spacecraft with the same inclination would have to deal with the same long term trend.

The challenge for LP was determining the long term trend of the spacecraft altitude since no prior spacecraft orbited the Moon in a low altitude polar orbit. There was a wide range of possible behavior predicted by propagating different gravity fields. Figure A-1 shows the actual behavior observed for LP once it was inserted into a circular orbit on Jan. 15, 1998 together with the predictions from 5 gravity fields determined prior to LP (Lun60d[7], the more recent Clementine based GLGM-2[8], Bills-Ferraris[9], Liu-Laing[10], and Ferrari et. al[11]) . The predictions from the LP based models (LP75G[3], LP100J[4]) match the actual observed altitude and so correctly incorporate the long term trend of LP. In addition, the altitude drop for the Apollo 16 subsatellite is shown. The model (Lun60d) that was used for mission design of LP provides the best prediction of any historic models. Follow-on JPL models to Lun60d prior to LP, e.g. Lun75a used by the Clementine navigation team or Lun75f (includes Clementine data but not LP), show similar propagations to Lun60d. If the actual gravity field behavior had been closer to that experienced by the Apollo 16 subsatellite, the LP mission would have had enough fuel to last at most 6 months without a possibility for a low altitude extended mission.

The Bills-Ferrari and GLGM-2 models show very similar structure in their predictions as shown by GLGM-2 in the enlarged view in Fig. A-2, but they diverge quickly from the actual orbit altitude about 15 days after epoch and go out of phase with the actual (i.e. altitude minimums correspond to actual maximums). Why the difference? The Lun60d and GLGM-2 models contain exactly the same data set (Lunar Orbiters I to V and Apollo 15 and 16 subsatellites) except Lun60d does not contain the Clementine data and both are roughly expanded to the same harmonic degree. However, subsequent JPL models with Clementine and no LP data are similar to Lun60d. Clementine has a 400-km periapse altitude and so only weakly senses the polar data. So the differences are probably due to different treatment of the LO-5 data set which had a periapse altitude of 100-km and some data noise problems. To find where the differences are coming from, the LP75G and GLGM-2 models were propagated for different truncations of the model (degree 10, 12, 14, 16). The propagations of the LP models have the same phase of GLGM-2 when truncated at roughly degree 10. So the differences in propagation are due to differences in the gravity coefficients from degree 10 and higher. However, when looking at the RMS differences of the gravity spectrums versus degree, there does not appear to be any large discrepancy beyond the associated uncertainties of the gravity fields.

After the third lunar orbit insertion burn on January 15, 1998, LP began orbiting the Moon in a circular 100-km orbit. However, to keep the altitude within a roughly 80 to 120 km range, orbit maneuvers were required about every 56 days or 2 lunar months and

4 sweeps of the orbit over a given region of the Moon (see Beckman and Concha's Table 2 for a listing of all maneuvers for the first 5 months of the mission[1]). We call each group of 56 days a cycle and each begins and ends with a face-on orbit plane as viewed from the Earth. The altitude history for each cycle of the nominal mission is displayed in Fig. A-3. The last 100-km cycle was extended 14 days to December 19, 1998 when the altitude was reduced to an average of 40-km.

The altitude of LP was reduced to 40-km (cycle-1 of the extended mission) to refine the gravity field in preparation for an even lower altitude. On January 29, LP was lowered to an average altitude of 30-km where it remains for the rest of the extended mission (to July 31, 1999). It was decided to not lower the altitude further due to the uncertainty in the terrain height and LP possibly impacting the surface. The altitude history for each orbit cycle in the extended mission is shown in Fig. A-4 (the altitude is with respect to a 1738-km sphere). The first three 28-day cycles at 30-km (cycles 2,3,4) have periapse on the nearside of the Moon, cycle-4 is a transition 14-day cycle, and then the next three 28-day cycles (cycles 6,7,8) have periapse on the farside of the Moon (cycles 7 and 8 have not been completed and are not shown at the time of this writing). Figures A-5 and A-6 show the minimum altitude of LP over the Moon's average surface of 1738-km for these two phases (nearside and farside). The lowest LP altitude or darkest shaded area corresponds to an altitude between 10 and 15 km. At times the altitude is about 7 or 8 km above the actual lunar surface especially over the south pole mountains and farside highlands.

LUNAR REFERENCE FRAME

The lunar gravity field was developed using the lunar orientation specified by JPL planetary ephemeris DE403. On the ephemeris, the orientation of the Moon with respect to the Earth Mean Equator of J2000 (EME2000) is given by three Euler angles[12]: (1) the rotation by angle φ about the Z-axis from the vernal equinox or X-axis of EME2000 to the intersection of the ascending node of the lunar equator, (2) the tilt up about the X-axis by θ to match the lunar equator, and (3) the rotation by ψ along the lunar equator to the lunar prime meridian. These three angles describe the lunar librations to a very high accuracy (2-3 cm accuracy for the Lunar Laser Ranging[13]) and were determined from numerically integrating the lunar orientation together with the planetary positions. These three angles give a lunar body-fixed coordinate system with axes aligned with the lunar principal axes.

All the results of the lunar gravity field presented in this paper use the orientation of DE403. If, however, one wishes to use the lunar gravity field with the IAU lunar pole and prime meridian, some corrections must be made. The IAU orientation (either IAU91[14] or IAU94[15]) is also a lunar body-fixed orientation with some lunar librations included but with the body-fixed axes specified by the mean-pole of the Moon. These axes are offset from the principal axes of DE403 by rotations using three small angles and amounts to about 700 meters at the lunar surface for two of the angles. The conversion from mean (M) axes of the IAU to the principal axes (P) is given by Williams[16] for DE245. The angles for DE403 change slightly and are

$$P = R_z(63.8986'')\, R_y(79.0768'')\, R_x(0.1462'')\, M\,.$$

These rotations can also be included in the right ascension ($\alpha = \varphi - 90^\circ$), declination ($\delta = 90^\circ - \theta$), and prime meridian ($W = \psi$) series of the IAU by adding more terms to the series. To convert the IAU series (either 91 or 94) to the principal axes used by DE403, add 0.0553 cos W_p + 0.0034 cos($W_p + \Omega$) to α, add 0.0220 sin W_p + 0.0007 sin($W_p + \Omega$) to δ, and add 0.01775 - 0.0507 cosW_p - 0.0034 cos(Wp+Ω) to W, where Ω =E1 of the IAU series and W_p is the polynomial part of W[17]. These terms come from spherical trigonometry relations for the three small rotations above. With the IAU series converted to the principal axes, the remaining differences between the DE403 coordinate frame and the IAU are due to truncation of the libration terms in the IAU series. Figures L-1 and L-2 show the magnitudes of the position differences in the body-fixed axes on the lunar surface of the corrected IAU91 and IAU94 coordinates, respectively, with the DE403 axes. The results of using the IAU91 axes amount to errors in lunar orientation of 440 meters during the LP nominal mission whereas the maximum errors from using IAU94 are 140 meters. As an alternative, the gravity coefficients could be rotated to the mean-pole axes from the principal axes but this is a more complicated procedure for the x and y axes rotations.

ORBIT DETERMINATION MODELING

Lunar Prospector was tracked nearly 24 hrs a day by the DSN's 26 meter (DSS 16, 46, 66) and 34 meter (DSS 24, 27, 34, 42, 54, 61) diameter tracking stations at a one second data sample rate with interruptions lasting as long as 45 minutes per orbit as a result of occultation by the Moon. Only two-way Doppler and range data were used in the orbit determination process at JPL and both types of data were compressed to ten second intervals. The compression was done to reduce the large volume of data and remove most of the spin signature from the Doppler data (<0.1 mm/s). The Doppler data are essentially a differenced range measurement. So for sample times that are integer multiple of the spin period of 5 seconds, the antenna phase center returns to nearly the same location. The Doppler (range change divided by sample time) is small. The spin of LP using the omni antenna for both the up-link and down-link results in a bias of (1+240/221)S Hz, where S is the spin period in revs/sec (0.2 for LP). The bias is thus 27.3 mm/s. For the medium gain antenna, the polarization changes for the down-link and the bias is (1-240/221)S Hz or -1.12 mm/s. For omni down-link, the one second data has a signature with a 5 second period and an 8.15 mm/s amplitude due to the spin. This indicates a 6.4 mm offset of the omni antenna phase center from the spin axis[2]. For a medium gain antenna down-link, the spin amplitude reduces to 4.5 mm/s. The S-band Doppler data were weighted with a sigma of 0.015 Hz or 1 mm/s for the nominal mission and 0.105 Hz or 7 mm/s for the extended mission. Similarly, the S-band range data were weighted with a sigma of 2 meters for the nominal mission and 14 meters for the extended mission.

The data were processed using JPL's Orbit Determination Program (ODP)[18]. The ODP solves for the spacecraft position, velocity, and other requested parameters using a square root information (SRIF) weighted least squares filter[19,20]. The determination of the gravity field LP100J was done on the Caltech/JPL HP SPP2000 supercomputer and the orbit fits presented in this paper were done on an HPJ282 workstation. A relatively simple estimation model was used because the orbit perturbations were well characterized -- simple spacecraft structure, no out gassing or momentum wheel dumps, a fixed solar pressure cross section and no atmospheric drag. There were six gravity perturbation models used in determining the orbit of LP. They were the newtonian point-mass model, the relativity corrections to the point-mass model, the direct and indirect oblateness models, the tide model, and the solar radiation pressure model. The newtonian point-mass model computes the gravitational acceleration of the spacecraft due to the nine planets, the Sun and the Moon by treating the bodies as point-masses. The direct oblateness model calculates the acceleration of the spacecraft due to the Moon's oblateness, as well as the acceleration of the spacecraft due to the Earth's oblateness. The lunar gravity model used was LP100J[4], a 100th degree spherical harmonic expansion using normalized coefficients[21]. The indirect oblateness model determines the acceleration of the Moon due to the oblateness of the Earth and the acceleration of the Moon due to its own oblateness interacting with the point-mass of the Earth. The relativity model computes the relativistic perturbative acceleration caused by the Sun, Earth, and Jupiter on the Moon. The solid tide model computes the tidal acceleration effect of the lunar tides caused by the Sun and the Earth. Finally, the solar radiation pressure model determines the acceleration due to solar radiation on the spacecraft in three directions (ecliptic south $\mathbf{U}_{By}$, Sun-to-spacecraft $\mathbf{U}_{Br}$, and the normal to the two in the ecliptic plane $\mathbf{U}_{Bx}$); LP is simply modeled as a bus spacecraft component. Figures O-1, O-2, and O-3 show the initial solar radiation pressure solutions with their respective 1-Sigma error bars for the first couple of months into the nominal mission. It can be seen in Fig. O-3 that there was an exponential decline in the acceleration in the south ecliptic direction; this may be due to spacecraft outgassing left over from the lunar orbit insertion burn and/or spacecraft radiation, which reduced as LP adapted to the harsh environment of space. A lunar albedo model[22] was also included, although for the 2-day arcs it is a small effect.

A number of other models were used in the ODP. To name a few, there were the relativistic light time correction model, the solid Earth tide correction model, the continental plate motion and ocean loading models, precession and nutation models, and a model for media calibrations for the data due to the Earth's ionosphere and troposphere. The *a priori* sigma for each of the spacecraft's position components was 2 km and was 2 m/s for each of the velocity components.

During the course of the mission, LP performed orbit correction maneuvers, reorientation maneuvers, and trimmed its spin rate. These maneuvers and spin rate trims were not modeled when processing a data arc; instead, data arcs end just before and begin just after such events occur to avoid corrupting the gravity field with unmodeled errors.

OVERLAP ORBIT ERRORS FOR THE NOMINAL MISSION

All the Doppler and range data from the LP mission as discussed above are included in the lunar gravity field in data segments or arcs about 2 days long (equivalent to 24 orbits about the Moon). The data arcs are time continuous - the end of one arc is the beginning of the next. The orbit determination errors are accessed by looking at the overlap differences between arcs or root-mean-square (RMS) position differences. For each arc, the spacecraft position, velocity, three orthogonal solar pressure coefficients, and range and Doppler biases are determined. The orbit is then propagated two hours to overlap one orbit with the next arc. So the overlap errors also include the errors from propagating one orbit but this contribution to the overall error is small. These overlap errors are the relative errors between arcs but the range in values are a good indication of the absolute orbit error since the gravity error does not generally affect successive arcs in the same manner due to a varying groundtrack over the Moon.

Figures N-1, N-2, and N-3 show the radial (R), transverse (T), and normal (N) overlap errors, respectively, for cycle-2 of the LP nominal mission. Cycle-2 was chosen to represent typical orbit errors because of the noted outgassing of the spacecraft in cycle-1. Both the previous 75th degree and order lunar gravity field LP75G[3] and the newer 100th degree and order LP100J model[4] orbit errors are displayed. The new gravity model LP100J shows significant orbit error reduction with radial errors typically better than one meter, and the transverse (in the orbit plane normal to the radial direction and roughly along the velocity direction) and normal (to the orbit plane) errors are mostly less than ten meters. There doesn't seem to be a strong correlation of the orbit error with spacecraft longitude and the amount of time LP is over the farside of the Moon.

The radial errors are very comparable to results of previous missions. Reconstruction of the Magellan orbits with a recent high resolution Venus gravity field (e.g., Konopliv et. al.[23]) with the purpose of redetermining the shape of Venus (Rappaport et. al.[24]) led to radial orbit errors also typically less than one meter from measuring overlaps of three day arcs (or 22 orbits). The errors in the other directions were higher than those of LP due to the increased parallax for LP. The typical Magellan orbit error in the transverse direction was 200 meters and in the normal direction was 50 meters with exceptions due to orbit geometry and superior conjunction. The recent overlap analysis of the circular Gravity Calibration Orbit (GCO) with the latest Mars gravity field model MGS75B for Mars Global Surveyor or MGS[25] shows radial errors again less than one meter, transverse errors of ten meters and normal errors of 100 meters. A two day arc length (or 24 orbits) was used for MGS and normal errors were larger due to the near edge-on geometry of the orbit plane as seen from the Earth (in this case the orbit plane normal is normal also to the line-of sight direction information from the Doppler measurements).

The orbit errors for the LP nominal mission are lower when compared to previous lunar missions. Orbit errors for previous lunar missions include Clementine where the overall error (R, T, and N together) was assessed to be about 100 meters[26] and the Apollo 15 subsatellite where the radial errors were about 5 meters and 200 meters in the T and N directions[7].

In another overlap test to get an idea of how fast the orbit error propagates, a 2-day solution arc was propagated 2 days plus a few hours past the end of the its data to overlap not with the next but with the third data arc (so one 2-day arc is skipped). The differences for the first and third arc for a one orbit overlap (2 hours) were computed in RTN coordinates and are displayed in Fig. N-4, N-5, and N-6. The orbit error after two days without tracking are about 1.5 meters for R, 30 meters for T and not much growth for N (still less than 10 meters). Due to the well determined gravity field, the lack of atmospheric drag, and limited outgassing for a spinning spacecraft, the orbit errors grow very slowly with time.

The resulting RMS of the Doppler and range observables from the two day solution fits are very near the actual data noise of the observations. Figures N-7 and N-8 show sample residual Doppler and range plots for an LP nominal mission arc fit with gravity model LP100J. Figures N-9 and N-10 show the Doppler and range residual RMS for the cycle-2 data arcs used for orbit overlap comparisons. The typical Doppler noise for the 10-second samples is 0.3 mm/s and the typical range noise is 0.5 meters. Again, the 10-

second Doppler sample is convenient since it has the spin signature mostly removed (<0.1 mm/s). There is still some small systematic gravity signature in the 100-km altitude data and further high resolution gravity determinations beyond degree 100 will probably reduce further the data RMS and orbit overlap errors.

COVARIANCE ORBIT ERRORS FOR 100-KM ALTITUDES

The orbit error for the LP nominal mission was also checked using a 40th degree truncation (for the purpose of computer run time reduction) of the full 100th degree and order gravity covariance of model LP100J. A 40th degree covariance will contain most of the orbit error as shown by the frozen orbit sensitivity (see Figure 23 of Konopliv et. al.[7]). Assuming negligible initial position and velocity error, the gravity error for an actual LP orbit at the beginning of the nominal mission (Jan. 15, 1998 where the geometry has the orbit plane face-on as viewed from the Earth) after a two day propagation amounts to a one meter radial error, 20 meter transverse error, and one meter normal error and is comparable to the overlap errors for the R and T directions but is smaller than the errors in the N direction. The small N error indicates the overlap error is mostly due to position and velocity error and not gravity error.

To quickly assess possible orbit error for other 100-km circular orbits with different inclinations, 40th degree error propagations were done for a near edge on circular orbit (as viewed from the Earth) that passes over the central farside for all inclinations. For this orbit eccentricity is small with $e=0.001$ so periapse altitude is initially 2-km below the 100-km average. The resulting maximum errors over a 2-day interval are shown in the R, T, and N directions in Fig. C-1 where again there is negligible initial position error. The errors show the lack of information in the lunar gravity field for inclinations of 40° to 80° where the maximum radial error is about 500 meters. This error would be substantially reduced if there were direct observations of the lunar farside gravity even from an LP type orbit. For the inclinations where we have previous missions ($<30^\circ$, and 90°) the R and N errors are about 20 meters. The large T error for all inclinations notes the need to fine tune the gravity field for orbits not previously included in the gravity field. LP never precisely has a circular edge-on orbit but instead has nonzero eccentricity pushing it tens of kilometers from circular.

Even in a polar near circular orbit (but more eccentric than above), the orbit error is very sensitive to the location of periapse. Again using the 40th degree covariance and assuming zero initial position and velocity uncertainty, the orbit error in the RTN coordinates were computed for a slightly eccentric polar orbit for difference locations of periapse. In this test $i=90^\circ$, $e=0.01$, periapse altitude is 80 km for a 100-km average altitude, and the geometry again has the orbit plane near edge-on as viewed from the Earth. The maximum R and N errors over a 2-day interval are shown in Fig. C-2 and the errors for T are in Fig. C-3. In all cases, the error when periapse is near the lunar farside equator is much greater than when periapse is on the lunar nearside (or argument of periapse ω is $-90^\circ<\omega<90^\circ$). Of course the gravity errors are greater for the farside because there is no direct observation of the farside gravity in LP100J or any other models. But also the periapse location through cycles 1-6 of the nominal mission favored the nearside and LP never follows precisely any of the orbits in Fig. C-1 and C-2. We also checked the errors for an orbit LP follows. For an LP orbit with periapse on the farside, the errors reduce to nearside values. For an LP data arc beginning March 27, 1998 at 22h, the mapping of the covariance showed 30 meter errors in T, less than a meter in R, and one meter in N after 2 days. This arc has periapse on the farside located at -60° latitude and with a periapse altitude of 91 km.

OVERLAP ORBIT ERRORS FOR THE EXTENDED MISSION

On December 19, 1998, the LP average altitude was reduced from 100-km to a 40-km average altitude and then on January 29, 1999 (41 days later) the mean altitude was reduced further to an average of 30-km above the surface. We have computed the overlap errors for the extended mission (or low altitudes) using the same technique and arc length described above for the nominal 100-km altitude nominal mission. Overlap errors were computed from the beginning of the 40-km altitude orbit to May 5, 1999 when the periapse of the orbit was transferred to the lunar farside.

Figures E-1, E-2, and E-3 show the overlap errors since the beginning of the 40-km orbit. The errors are significantly increased over the nominal mission with the radial errors being tens of meters and the transverse and normal errors being hundreds of meters. Most of the error is from truncating the gravity field solution at degree 100 although some of the error is from the data not being included in the LP100J gravity model (LP100J contains data to Feb. 8, 1999 or day 51 on the figures). With further high resolution models being developed (hopefully, degree ~180), the overlap errors will be significantly reduced. Note that the errors for the 40-km orbit are much less (up to day 41) than the 30-km orbit.

The error growth for 2 days were also computed for the extended mission using the same method as for the nominal mission discussed above. The 2-day overlap errors for the R, T, and N directions are also shown in Fig. E-1, E-2, and E-3 together with the overlap errors from only a 2 hour propagation. There is some increase in the errors with the average error increasing by 64%, 90%, and 53% for the R, T, and N directions, respectively.

Figures E-4 and E-5 show the residual RMS of the orbit fits for both the Doppler and range data. At the low altitudes, the RMS (~15 mm/s) is 50 times the actual data noise (~0.3 mm/s) for the Doppler and somewhat less for the range (RMS is ~15m). Although there are times with large data noise in the Doppler near the polar regions due to the increased background noise of the lunar surface when the spacecraft is at a lower altitude. Nevertheless, this is an indication of the large amount of high frequency (> degree 100) gravity information left in the Doppler residuals. Future higher resolution models will resolve small craters that are tens of kilometers in size.

LONG TERM ORBIT PREDICTIONS

To get an idea of how well the gravity field predicts the orbit of LP over a longer time, an initial 2-day solution arc was propagated 36 days and compared with the individual 2-day solutions over that time span. This was done for two cases during the nominal mission at 100-km where no spacecraft maneuvers were performed for that length of time (beginning May, 20, 1998 in cycle-3 and August 18, 1998 in cycle-5). Doppler data during these times are not in the LP75G model but similar orbit profiles are from cycles 1 and 2. The LP100J model contains data during this time. Table 1 shows the orbit differences with the 2-day solutions at the end of the 36-day propagation for the two gravity fields LP75G and LP100J. The initial orbits used for the propagation were also determined by the corresponding gravity field. The long-term errors are mostly in the alongtrack direction (T) and are several kilometers for LP100J and tens of kilometers for LP75G. The normal (N) errors do not grow with time in contrast to R and T and are due to initial orbit error and not gravity propagation error as mentioned previously.

FROZEN ORBITS

Of interest to future missions to the Moon is the long-term behavior of the orbit and the amount of fuel that will be required to maintain a near circular orbit (±20 km about a 100-km average altitude). Based upon the gravity model Lun60d[7], LP was designed for a fuel usage of 0.22 m/s per day plus 0.03 m/s per day margin (0.25 m/s per day total). The actual fuel usage has been very close to this value (within 10%) and surprisingly so, considering the wide range of values predicted by various gravity models (from near zero to greater than 1 m/s per day). The fuel usage experienced by LP can be reduced for future missions by moving the inclination of the orbit from being exactly polar as LP.

A frozen orbit is an orbit with no long term changes in eccentricity, argument of periapse, and inclination and so require minimum or no orbit maintenance burns. Using the same formulation as discussed in Konopliv et. al.[7] with the zonal gravity coefficients, the frozen orbit periapse altitudes for gravity models Lun60d and LP100J are displayed in Fig. F-1 together with the periapse altitude uncertainty from the zonal coefficients covariance matrix including correlations. The uncertainties are three times the formal to give a more realistic error. Based upon the latest data, the frozen orbit location at 100-km has shifted from 82.5° in Lun60d to closer to 85° in LP100J. Again, evident in the uncertainties is the lack of information for inclinations between 40 and 80 degrees.

CONCLUSIONS

Prior to being inserted into lunar orbit, the behavior of LP's orbit was unknown due to the large uncertainty of the gravity field. Once in orbit, the actual observed LP altitude was compared to five predicted altitude trends made by five historical gravity field models. The prediction from Lun60d was closest to the actual altitude observed and the actual fuel usage was very close to that assumed for the mission. A new lunar gravity field model (LP100J) of degree and order 100 was developed using the historical mission data, LO I-V and Apollo 15 and 16 as described by Konopliv et. al[7], and the more recent Clementine[8] data together with the data from Lunar Prospector's nominal mission and a few weeks of the extended mission. Gravity model validation of LP100J was performed by accessing orbit overlap errors. For the nominal mission at 100-km, the average orbit overlap errors between adjacent arcs were less than 1 meter in R, and less than 10 meters in T and N and is significantly less than previous gravity models. The errors appear to be independent of orbit geometry. For the low altitude extended mission, the average overlap errors between adjacent arcs were less than 15 meters in R, and less than 120 meters in T and N. The use of a higher resolution model (>100th degree) that has yet to be developed will significantly reduce these errors. Two initial 2-day solution arcs, with different epochs, were propagated 36 days to evaluate the long term periapse altitude predictions; at first using the LP75G gravity field, then using the LP100J gravity field. The predictions were compared with the solutions at the end of the propagation. In both cases, the propagations using the LP100J model resulted in much lower orbit errors of less than 100 meters in R and N and several kilometers in T. In addition, the location of a circular frozen orbit at 100-km has shifted from 82.5° inclination for Lun60d to about 85° for LP100J; a change that is of interest to future missions such as Selene.

ACKNOWLEDGMENTS

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REFERENCES

1. Beckman, M. and M. Concha, Lunar Prospector Orbit Determination Results, AIAA paper 98-4561, AIAA/AAS Astrodynamics Specialists Conference, August 10-12, Boston, MA, 1998.

2. Weischede, F., E. Gill, O. Montenbruck, Lunar Prospector Orbit Determination and Gravity Modeling Based On Weilheim 3-Way Doppler Measurements, 14th International Symposium on Space Flight Mechanics, Iguassu, Brazil, Feb. 8-12, 1999.

3. Konopliv, A.S., A.B. Binder, L.L. Hood, A.B. Kucinskas, J.G. Williams, Improved Gravity Field of the Moon from Lunar Prospector, *Science* 281, 1476-1480, 1998.

4. Konopliv, A.S. and D.N. Yuan , Lunar Prospector 100th Degree Gravity Model Development, *Lunar Planet Sci. Cong, 30th*, March 15-19, 1999.

5. Web site for Lunar Prospector is http://lunar.arc.nasa.gov

6. Swanson, D., Lockheed Martin, personal communication, 1999.

7. Konopliv, A.S., W. L. Sjogren, R. N. Wimberly, R. A. Cook, and A. Vijayaraghavan, in *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, Victoria, BC, 1993, edited by A. K. Misra, V. J. Modi, R. Holdaway, and P. M. Bainam (Univelt, San Diego, CA, 1993), pp. 1275-1294.

8. Lemoine, F.G., D.E. Smith, M.T. Zuber, G.A. Neumann, and D.D. Rowlands, A 70th Degree Lunar Gravity Model (GLGM-2) from Clementine and Other Tracking Data, *J. Geophys. Res.* 102, 16339-16359, 1997.

9. Bills, B. G., and A. J. Ferrari, "A Harmonic Analysis of Lunar Gravity," *Journal of Geophysical Research,* Vol. 85, No. B2, pp. 1013-1025, 1980.

10. Liu, A. S., and P. S. Laing, "Lunar Gravity Analysis from Long Term Effects," *Science*, Vol. 173, pp. 1017-1020, 1971.

11. Ferrari, A. J., W. S. Sinclair, W. L. Sjogren, J. G. Williams, and C. F. Yoder, "Geophysical Parameters of the Earth Moon Systems," *Journal of Geophysical Research,* Vol. 85, No. B7, pp. 3939-3951, 1980.

12. Newhall, XX and J.G. Williams, Estimation of the Lunar Physical Librations, *Celest. Mech. & Dyn. Astron.* 66, 21-30, 1997.

13. Dickey, J.O., P.L. Bender, J.E. Faller, XX Newhall, R.L. Ricklefs, J.G. Ries, P.J. Shelus, C. Veillet, A.L. Whipple, J.R. Wiant, J.G. Williams, C.F. Yoder, Lunar Laser Ranging: A Continuing Legacy of the Apollo Program, *Science* 265, 482-490, 1994.

14. Davies, M.E., V.K. Abalakin, A. Brahic, M. Bursa, B.H. Chovitz, J.H. Lieske, P.K. Seidelmann, A.T. Sinclair, and Y.S. Tjuflin, Report of the IAU/IAG/COSPAR Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 1991, *Celes. Mech. and Dyn. Astron.* 53, 377-397, 1992.

15. Davies, M.E., V.K. Abalakin, M. Bursa, J.H. Lieske, B. Morando, D. Morrison, P.K. Seidelmann, A.T. Sinclair, B. Yallop, and Y.S. Tjuflin, Report of the IAU/IAG/COSPAR Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 1994, *Celes. Mech. and Dyn. Astron.* 63, 127-148, 1996.

16. Williams, J.G., XX Newhall, and J.O. Dickey, Lunar Moments, Tides, Orientation, and Coordinate Frames, *Planet. Space Sci.* 44, No. 10, 1077-1080, 1996.

17. Williams, J.G., personal communication as expressed in a memo dated August 30, 1994 and a letter to Merton Davies October 25, 1994.

18. Moyer, T. D., Mathematical Formulation of the Double-Precision Orbit Determination Program (DPODP), Technical Report 32-1527 (internal document), Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, May, 1971.

19. Bierman, G. J., *Factorization Methods for Discrete Sequential Estimation*, Academic Press, New York, 1977.

20. Lawson, C. L., and R. J. Hanson, *Solving Least Squares Problems*, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1995.

21. Kaula, W. M., *Theory of Satellite Geodesy*, Blaisdell, Waltham, MA, 1966.

22. Floberghagen, R., P. Visser, F. Weischede, M. Vasile, On the Analysis of Lunar

Albedo Effects on Low Lunar Orbit and Gravity Field Determination, in *Proceedings of AAS/GSFC 13th International Symposium on Space Flight Dynamics*, Volume 1, GSFC, NASA/CP-1998-206858/VOL1, May 11-15, 1998.

23. Konopliv, A.S., W.B. Banerdt, and W.L. Sjogren, Venus Gravity: 180th Degree and Order Model, *Icarus* 139, 3-18, 1999.

24. Rappaport, N.J., A.S. Konopliv, A.B. Kucinskas, and P.G. Ford, An Improved 360 Degree and Order Model of Venus Topography, *Icarus* 139, 19-31, 1999.

25. Sjogren, W.L., D.N. Yuan, A.S. Konopliv, Mars Gravity Field Modeling with Mars Global Surveyor, 1999 AGU Spring Meeting, Boston, MA, May 31 - June 4, 1999.

26. Lemoine, F.G., D.E. Smith, M.T. Zuber, D.D. Rowlands, and G.A. Neumann, Results from the Clementine Lunar Geodesy Investigation, *Astrodynamics* 90, 201-233, 1995.

Table 1: Nominal Mission 36-Day Propagation Errors

Gravity Field	R-Error (m)	N-Error (m)	T-Error (km)
Epoch: May 20, 1998			
LP100J	15	20	2.0
LP75G	330	300	21.7
Epoch: August 18, 1998			
LP100J	64	10	5.5
LP75G	580	60	35.7

Figure S-1: Lunar Prospector Spacecraft

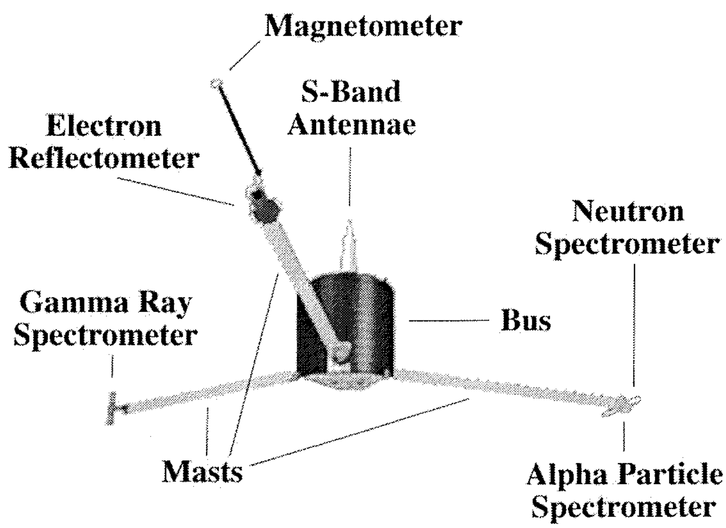

Figure A-1: Predicted Orbit Behavior for LP

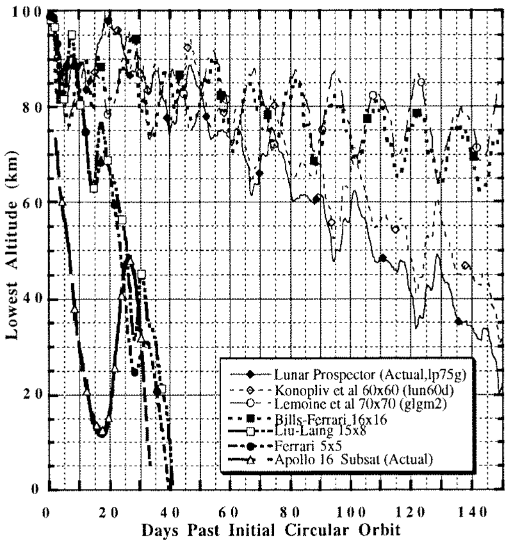

Figure A-2: LP Altitude at Periapse Predicts

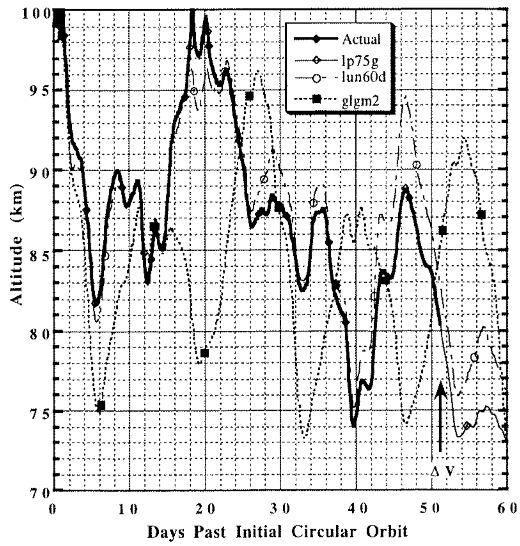

Figure A-3: LP Nominal Mission Periapse Altitudes

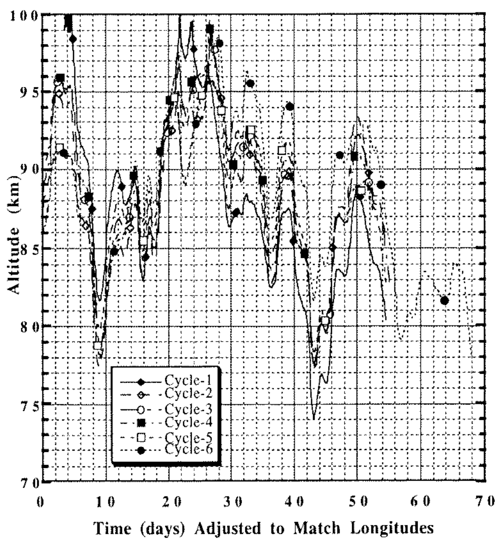

Figure A-4: LP Extended Mission Periapse Altitudes

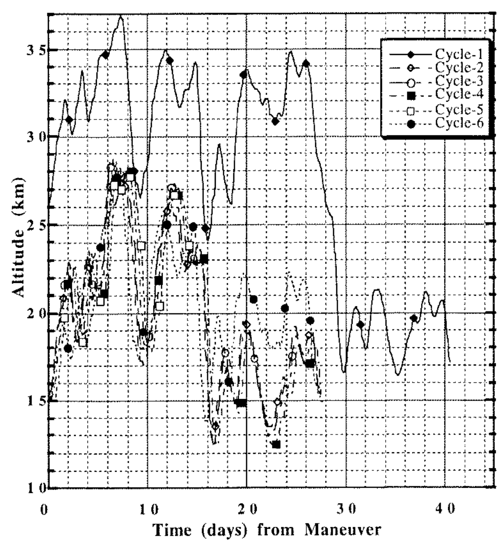

Figure A-5: LP Extended Mission Minimum Altitude
Nearside Phasing Orbit

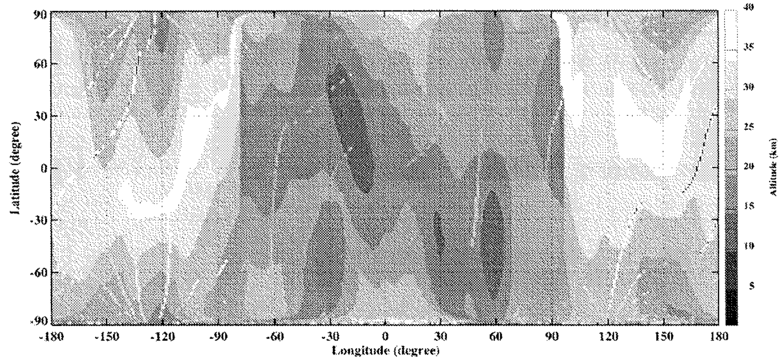

Figure A-6: LP Extended Mission Minimum Altitude
Farside Phasing Orbit

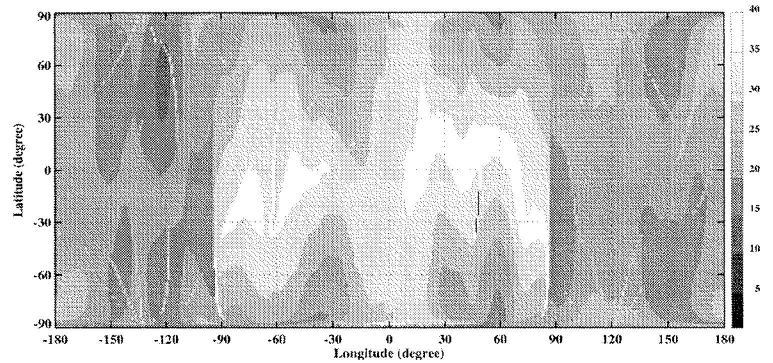

Figure L-1: Differences of Lunar Axes from DE403 Integrated Lunar Librations and 91 IAU Mean-Pole

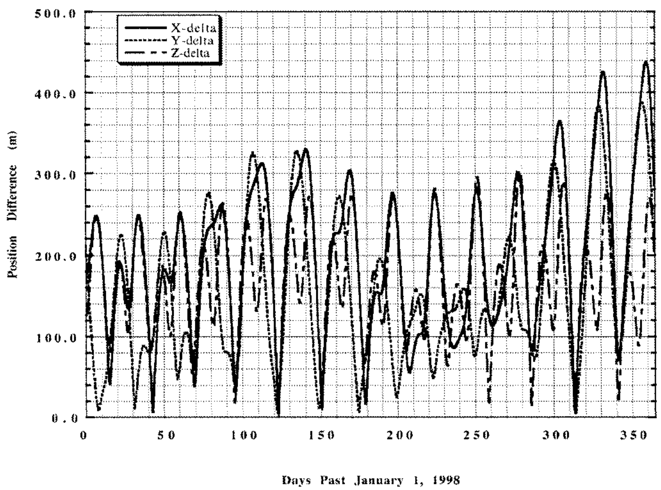

Figure L-2: Differences of Lunar Axes from DE403 Integrated Lunar Librations and 94 IAU Mean-Pole

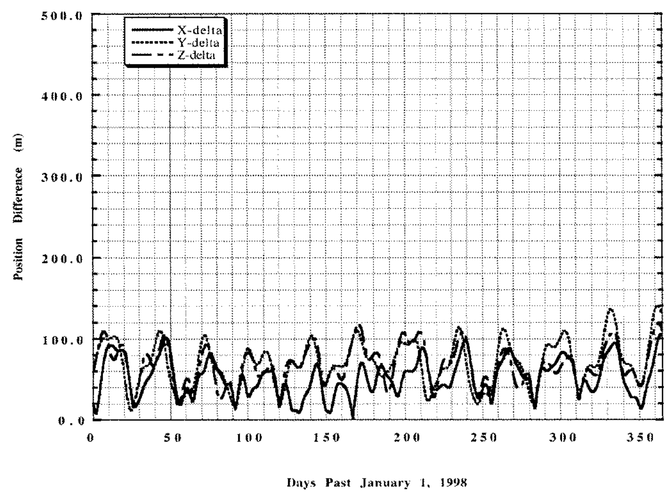

Figure O-1: Initial Solar Radiation Pressure Solutions ($U_{B\,r}$)

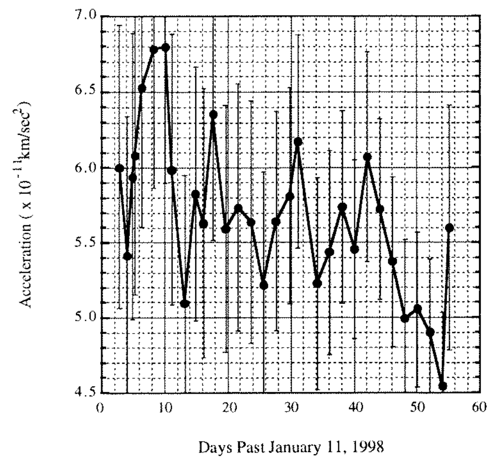

Figure O-2: Initial Solar Radiation Pressure Solutions ($U_{B\,x}$)

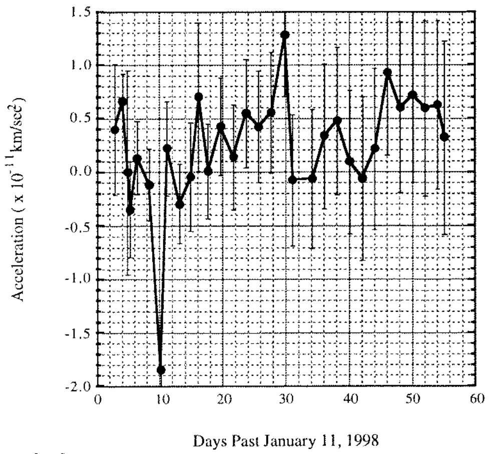

Figure O-3: Initial Solar Radiation Pressure Solutions ($U_{B\,y}$)

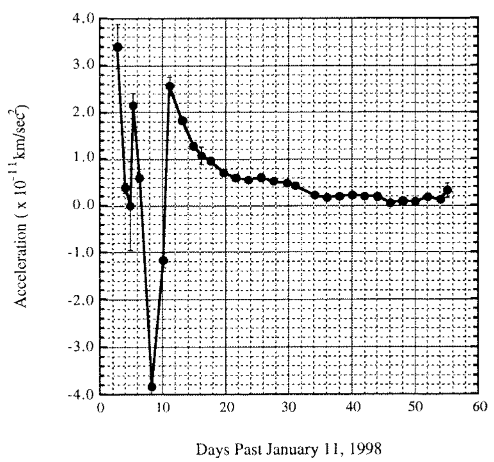

Figure N-1: Nominal Mission RMS Orbit Error (Radial)

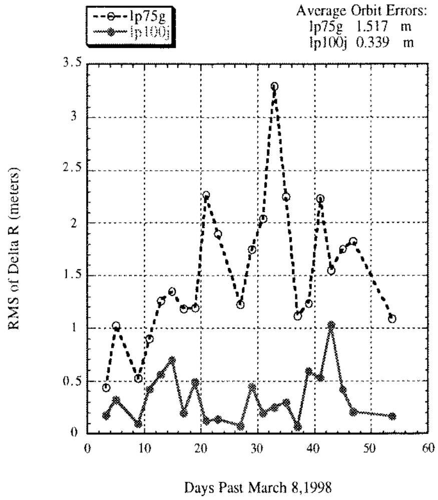

Figure N-2: Nominal Mission RMS Orbit Error (Transverse)

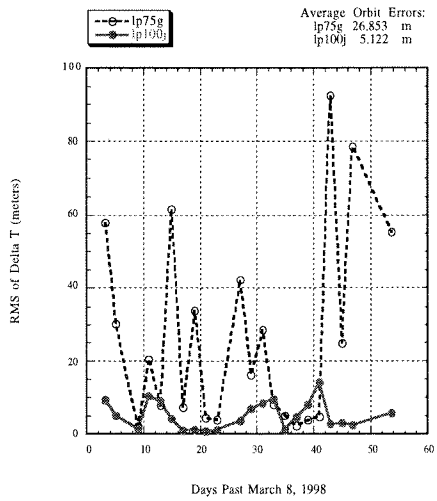

Figure N-3: Nominal Mission RMS Orbit Error (Normal)

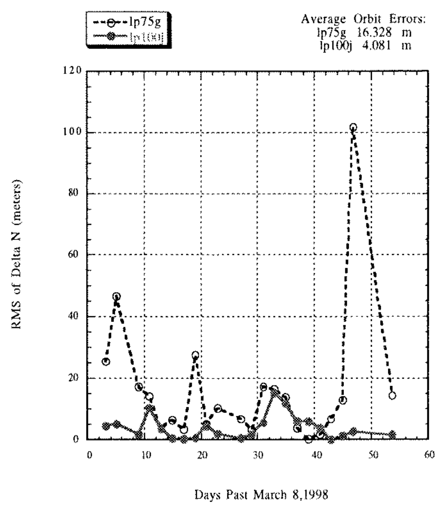

Figure N-4: Nominal Mission RMS 2-Day Orbit Error (Radial)

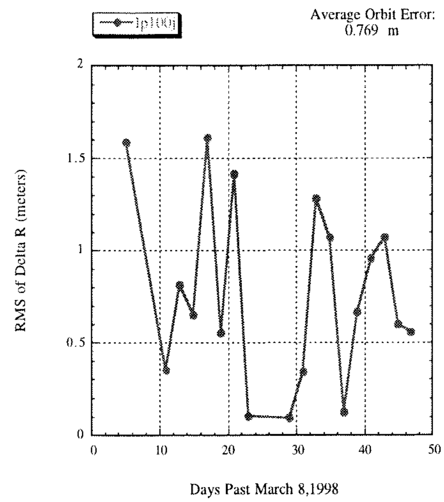

Figure N-5: Nominal Mission RMS 2-Day Orbit Error (Transverse)

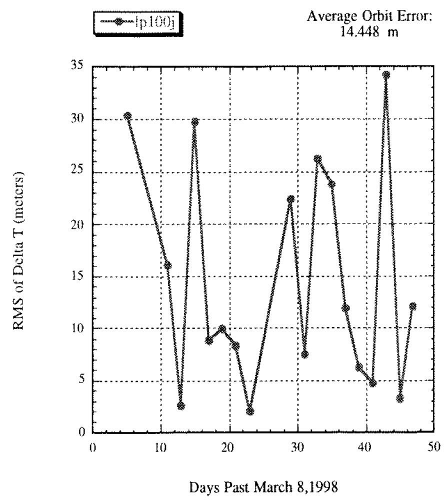

Figure N-6: Nominal Mission RMS 2-Day Orbit Error (Normal)

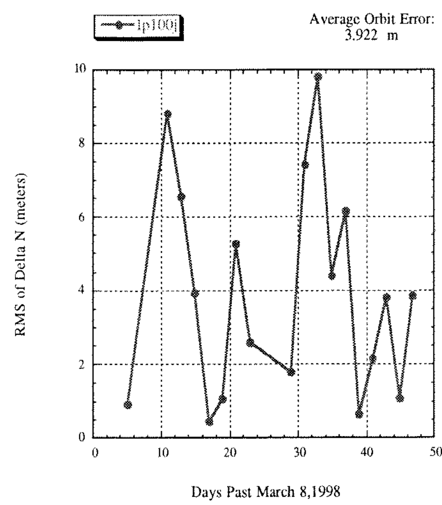

Figure N-7: Doppler Residual Sample

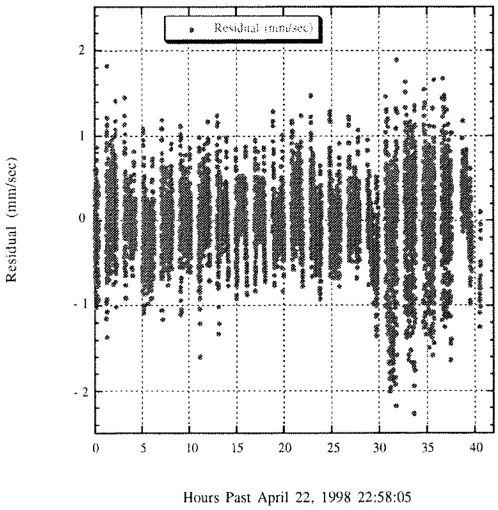

Figure N-9: Doppler Residual Summary (Nominal Mission)

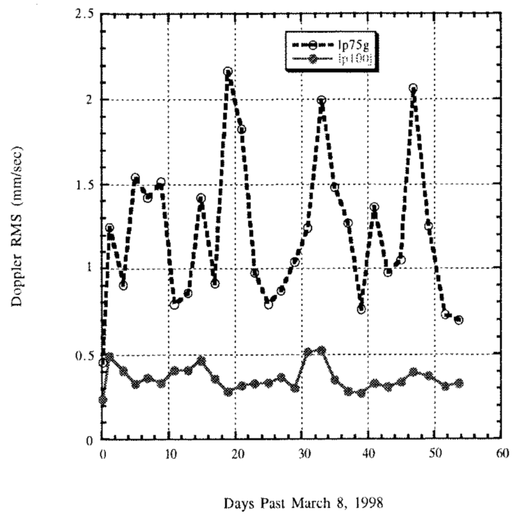

Figure N-8: Range Residual Sample

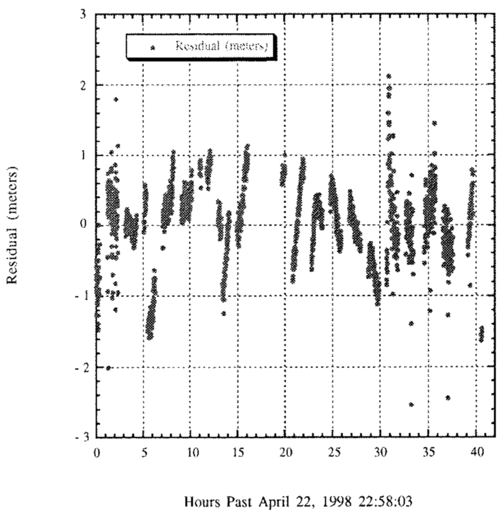

Figure N-10: Range Residual Summary (Nominal Mission)

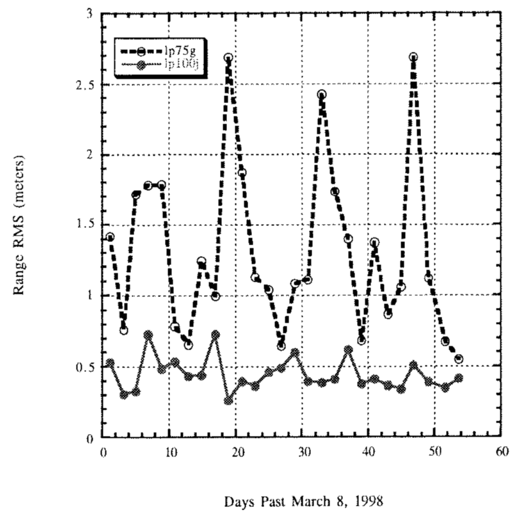

Figure C-1: Sensitivity of 100-km Circular Orbits to Inclination

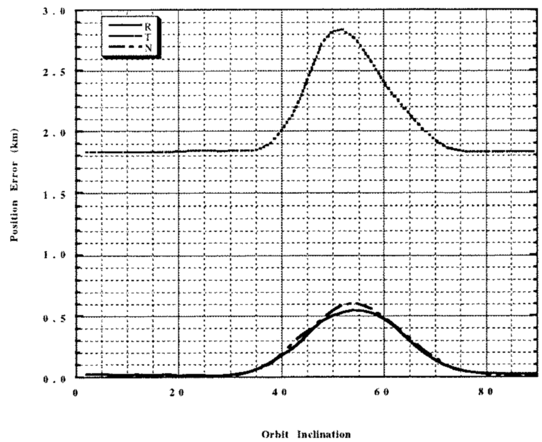

Figure C-2: Sensitivity of Polar Orbit Error to Periapse Location Radial and Normal Directions

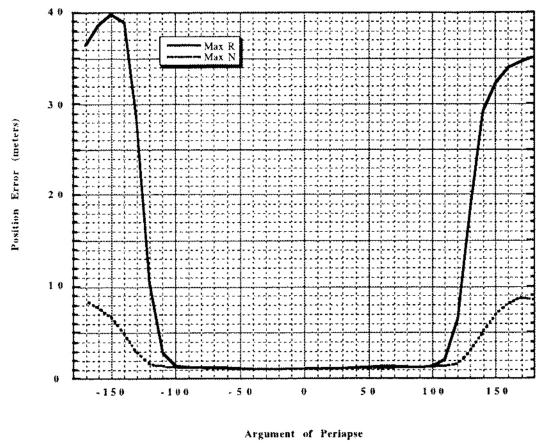

Figure C-3: Sensitivity of Polar Orbit Error to Periapse Location Transverse Direction

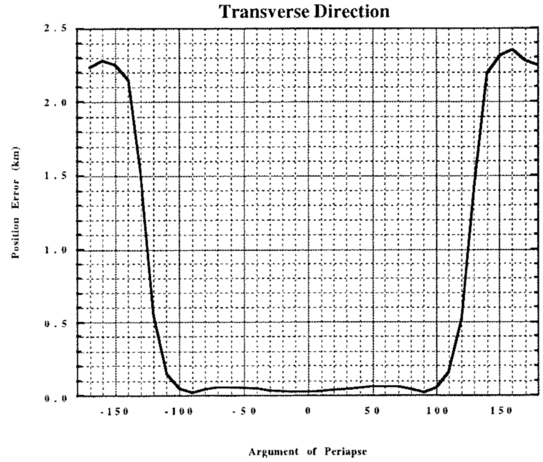

Figure E-1: Extended Mission RMS Orbit Error (Radial)

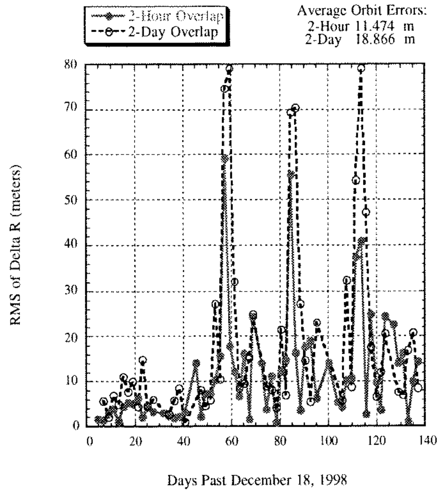

Figure E-4: Doppler Residual Summary (Extended Mission)

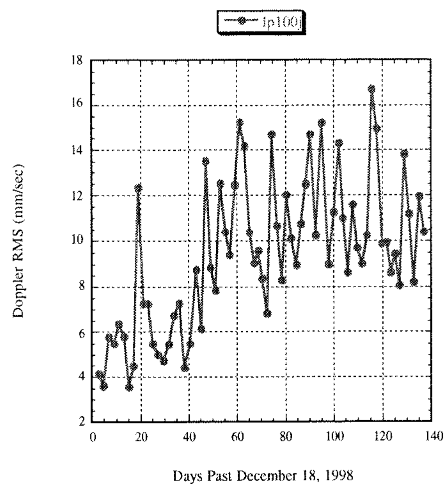

Figure E-2: Extended Mission RMS Orbit Error (Transverse)

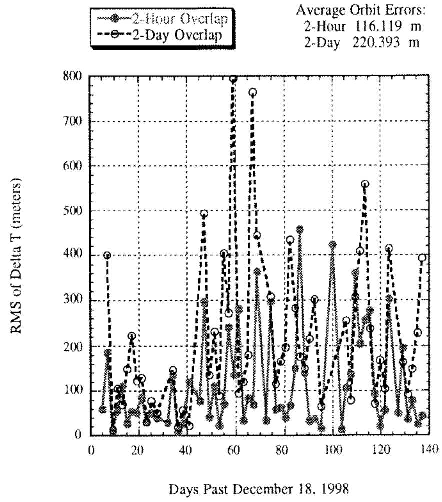

Figure E-5: Range Residual Summary (Extended Mission)

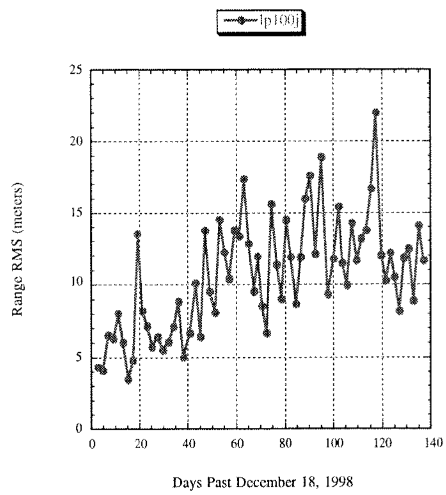

Figure E-3: Extended Mission RMS Orbit Error (Normal)

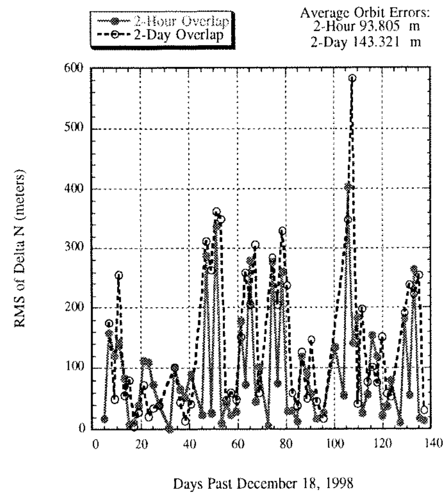

Figure F-1: Frozen Periapse Altitude for 100-km Orbits